Discovering Patterns Perfectly Matching the Data

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Scope

- In science and business we often need to find a formula (also called pattern) describing the relation between a dependent variable and a set of independent variables
- Once found, such formula could be very useful in describing the relation between its input and output parameters
- The main purpose of this white paper is to show a simple method to find such formula using linear algebra
- We'll show that with few (X, Y) pairs it is always very easy to find a formula capable to perfectly match the given data
- As example we'll use the (X, Y) pairs from table 1.

Table 1

X	Y
1	1
2	3
3	5
4	7

Formalizing the Pattern Design

When trying to find a mathematical pattern what we look for is a function F which generates the values of Y from the values of X in such a way as they match the given combinations (X_1,Y_1; X_2,Y_2; X_3,Y_3; X_4,Y_4).

Mathematically the relation can be written as:

$$Y_1 = F(X_1);\ Y_2 = F(X_2);\ Y_3 = F(X_3);\ Y_4 = F(X_4)$$

There are numerous ways to design the function F however, in general for few pairs of data points the easiest and safest way is to use linear algebra.

PATTERN DESIGN ALGORITHM

Generating Matrix

1. Select a set of independent functions (called base).

   In the examples below we'll use polynomial functions:

   [A]: $X^0, X^1, X^2, X^3, \dots$

   [B]: $X^0, X^2, X^3, X^4, \dots$

   [C]: $X^0, X^3, X^4, X^5, \dots$

   Many other functions can be used, common examples are based on but not limited to the exponential and trigonometric formulas.

2. Create the generating matrix M using a number of functions equal to the number of original X,Y pairs.

   For the polynomial functions in example [A]:

$$M = \begin{bmatrix} X_1^0 & X_1^1 & X_1^2 & X_1^3 \\ X_2^0 & X_2^1 & X_2^2 & X_2^3 \\ X_3^0 & X_3^1 & X_3^2 & X_3^3 \\ X_4^0 & X_4^1 & X_4^2 & X_4^3 \end{bmatrix}$$

Where X_1, X_2, X_3, X_4 are the four given values of X. In our approach the independent variables X are embedded in the generating matrix M.

A Little Algebra

3. Create the output vector as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

4. Compute the coefficients [C]:

Here $[]^{-1}$ represents the inverse of the generating matrix M. In our algorithm M is a square matrix so to be invertible it's determinant must be different from zero.

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} X_1^0 & X_1^1 & X_1^2 & X_1^3 \\ X_2^0 & X_2^1 & X_2^2 & X_2^3 \\ X_3^0 & X_3^1 & X_3^2 & X_3^3 \\ X_4^0 & X_4^1 & X_4^2 & X_4^3 \end{bmatrix}^{-1} * \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

5. The pattern generating formula becomes:

Where M is recreated for any set of X and C is from now on a constant vector.

$$Y = M * C$$

EXAMPLES

Example: Excel implementation [example A]

1. Enter the original pairs in cells B4-C7

Table 2

	B	C
3	**X**	**Y**
4	1	1
5	2	3
6	3	5
7	4	7

2. Create the generating matrix in cells B14-E17 by typing in cells B14-E14 the formulas *=B3^0*, *=B3^1*, *=B3^2*, *=B3^3*, then select cells B14-E14 and pull down the lower right corner by 3 cells.

Table 3

	B	C	D	E
14	1	1	1	1
15	1	2	4	8
16	1	3	9	27
17	1	4	16	64

3. Compute the inverse M^{-1} by typing in cell B26 the formula *=MINVERSE(B14:E17)*, select cells B26-E29, press key F2 followed by Ctrl-Shift-Enter.

Table 4

	B	C	D	E
26	4.000	-6.000	4.000	-1.000
27	-4.333	9.500	-7.000	1.833
28	1.500	-4.000	3.500	-1.000
29	-0.167	0.500	-0.500	0.167

Example: Excel implementation [A] - cont

4. Compute the coefficients C by typing in cell B31 the formula =*MMULT(B26:E29,C4:C7)*, select cells B31-B34, press key F2 followed by Ctrl-Shift-Enter.

Table 5

	B
31	-1
32	2
33	0
34	0

2. From C create the engendering formula:

$$Y = -1 + 2 * X$$

Cells B31-B34 contain the coefficient of X^0, X^1, X^2, and X^3.

3. For any value of X compute the corresponding Y by replacing X in the formula above. For example:

X = 8 implies Y=-1+2*8=15

Examples of patterns

Table 6 below shows several examples of patterns found using the Excel implementation.

Table 6

X	Ya	Yb	Yc	Yd
1	1	1	1	1
2	3	3	3	3
3	5	5	5	5
4	7	7	7	7
5	9	9.96	15.506	716.73
6	11	15.80	48.590	8707.58
7	13	27.40	140.952	58473.76
8	15	48.60	348.976	282983.99

The engendering formulas are:

$$Ya = 2 * X - 1$$

$$Yb = -0.04 + 1.4 * X^2 - 0.4 * X^3 + 0.04 * X^4$$

$$Yc = 0.4458 + 0.8735 * X^3 - 0.3614 * X^4 + 0.042168 * X^5$$

$$Yd = 0.9095 + 0.0945 * X^5 - 3.9638 * 10^{-3} * X^8 + 4.0352 * 10^{-5} * X^{11}$$

Example: R implementation [A]

1. Enter the initial pairs

2. Create the generating matrix

3. Find the constants C

   It produces the same values as in Excel

4. From the model compute Y for X in the range 1-8

   The result are: 1, 3, 5, 7, 9, 11, 13, 15

```
> X <- matrix(c(1,2,3,4),ncol=1)
> Y <- c(1,3,5,7)
>
> M <- cbind(X^0,X^1,X^2,X^3)
>
> C=solve(M,Y)
> C
[1] -1  2  0  0
>
> XI <- matrix((1:8), ncol=1)
> MI <- cbind(XI^0,XI^1,XI^2,XI^3)
> YI <- MI%*%C
```

Example: R implementation (cont.)

Using the R code above we can recreate the same patterns found with Excel, and sometimes experiment with much more complex bases or even arbitrarily chosen functions.

- As a generalization let's choose the next pair as (5, 7) and the base: $X^{0.31}$, $X^{1.54}$, $X^{3.57}$, $X^{3.78}$, $X^{9.11}$. The engendering formula becomes:

$$Y = -0.4356 * X^{0.31} + 1.564 * X^{1.54} - 0.41086 * X^{3.57} + 0.2818 * X^{3.78} - 2.576 * 10^{-6} * X^{9.11}$$

- Another example uses sinusoidal functions:

$$Y = 4.7471 * Sin(X) - 21.6737 * Sin(2 * X) - 43.5023 * Sin(3 * X) - 30.1958 * Sin(4 * X)$$

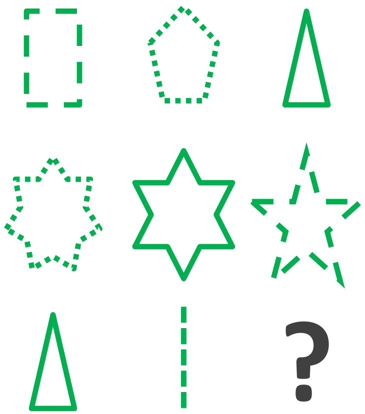
CONCLUSIONS

Mathematical Perspective

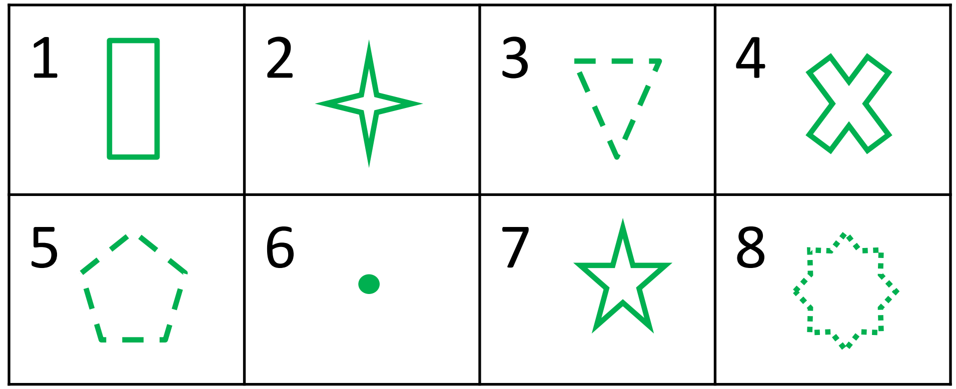
- It is possible and generally easy to find a formula describing the relation between the values of data pairs using simple algebra.
- The discovered pattern is not unique and in most cases it is possible to find a matching formula using almost any type of base and even almost any type of function, not only polynomial.
- It could be easily shown that almost any value could be assigned to the next pair of data.
- The algorithm can be easily expanded for several independent variables.
- If the number of pairs becomes very large this method involves tedious matrix manipulations and the result is too complex to have much practical value.

Qualitative Patterns

Test Set



Options Set



- Most qualitative patterns can be translated into numerical patterns.
- Steps to solve a Test Set: find relevant features, convert features to numbers, find numeric patterns, predict next values, translate numbers into features.
- To avoid the multiple patterns issue tests give an Options Set
- Here the relevant features are number of vertices and line type.

Alternate Reality?

- Since values predicted by each model are so different inside and outside the original range of X it is evident that, at best, a single model is correct although several models can be acceptable approximations.
- This Alternate Reality issue is well known in other fields:
  - Immanuel Kant suggested a division between the natural world called *Noumena* (our X,Y pairs) and the human vision of it called *Phenomena* (model M).
  - Klaus Conrad suggested the term *Apophenia* to describe a "*specific experience of an abnormal meaningfulness*" or our tendency to see patterns in random data.
- To dispel any doubts, the pairs used in this example are totally independent being created by a quasi-random number generator from R (see code snippet)

```
> set.seed(8555)
> C <- floor(t(runif(10))*10)
> Y <- C[1:4]
```

Final Word

- The power and beauty of mathematics could work for or against us depending on how we handle it's intricacies.
- Are you sure you are not gambling the future of your company with the latest analytic fad?
- Contact use today to learn how we can help you validate your analytic processes.

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