Measuring Financial Risk – Part 1

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In this white paper will show some of the various traditional measures of financial risk and comparisons with the prediction based on the standard definition of risk (defined as the sum of all products between the potential outcome and the probability of that outcome, the summation being done over the entire space of possible outcomes – *Hubbard 2009*).

In what follows we'll adopt the risk neutral perspective and the categorization of knowledge into the known, the unknown, and the unknowable (*Gomory, 1995*). To explain these concepts in a way easy to understand let's imagine that there is an urn filled with balls and a person interested to estimate the chance of extracting a black ball. In the case of known a person is aware that the urn has 5 red and 3 black balls. In the case of unknown the person knows only that the urn has red and black balls. As for the case of unknowable the person doesn't even know the number of colors or the balls that may be inside.

In this first part we'll assume that in all cases under analysis we are dealing with entirely known events meaning that we know both the cost of each event and its probability, or to be precise the Probability Distribution Function (PDF) is completely specified or it could be accurately derived.

A common traditional measure of the financial risk is based on the Standard Deviation of the investment returns described by the formula below (unbiased estimator of the population variance with the Bessel correction):

$$S = \sqrt{\frac{1}{N-1} \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2} \qquad (1)$$

Where x_i represent the samples, N the total number of samples, and $\bar{x}$ the average of samples determined as:

$$\bar{x} = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i \qquad (2)$$

Or as Markowitz, the father of portfolio theory, wrote (*Markowitz, 1952*) *"the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing"*.

Some of the more sophisticated measures of risk or volatility are now derived from the ARCH (Auto Regressive Conditional Heteroskedasticity) models. However since the main idea is still based on the assumption that the variance is the measure of volatility these new models don't bring a fundamentally new view. So even if there is no explicit assumption of normality, by neglecting the effect of higher level statistical moments (skewness, kurtosis, etc) in fact we implicitly assume that the investment return has a normal distribution.

By taking as example the returns of Microsoft (MSFT) stock over the last decade (from Aug 1st, 2000 to Aug 1st, 2010) we could determine the statistical parameters shown in the table below.

Table 1

Minimum	$15.15
Maximum	$37.06
Average	$26.58
Standard Deviation	$3.181
Skewness	-0.420
Kurtosis	1.443

Assuming the distribution is normal then the range of possible returns over the entire period under analysis could be described as below:

$$[m - n \cdot \sigma, m + n \cdot \sigma] \quad (3)$$

Where $\bar{x}$ represents the average and σ the standard deviation of daily returns (approximated by S). The coefficient n could be derived from the probability of having the real return within the desired range.

Table 2

Probability	68.3%	95.4 %	99.7%	99.994%	99.99994%	99.9999998%
n	1	2	3	4	5	6
Minimum	\$23.39	\$20.21	\$17.03	\$13.85	\$10.67	\$7.49
Maximum	\$29.76	\$32.94	\$36.12	\$39.30	\$42.49	\$45.67

Assuming the distribution is normal the probability of a return for example under \$21 is 3.9% and for a return under \$15.15 is 0.016%.

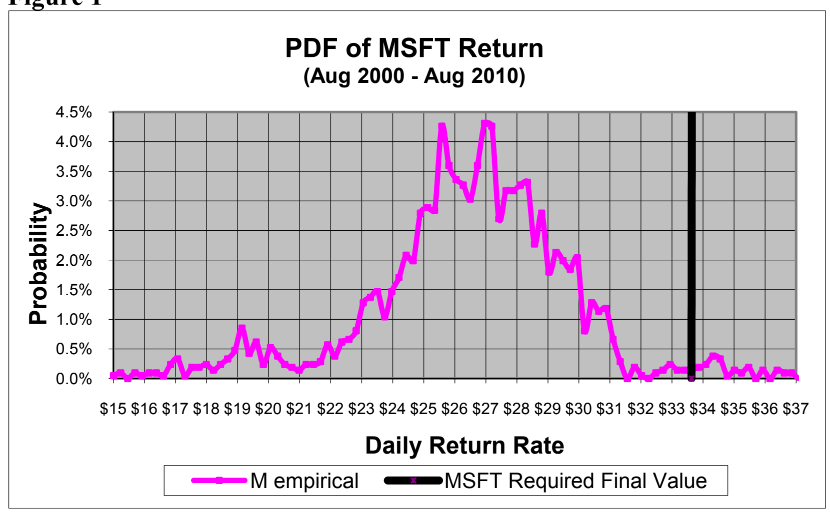
With a probability of 1% the end value of this investment is \$19.18 which implies that the Value at Risk VaR (*Jorion 2006*) is \$26.58 - \$19.18 = \$7.40.

From Table 2 aTable 1bove it is obvious that returns don't follow the normal distribution at least based on the kurtosis' value. This is the reason why it makes sense to use a more generic distribution to account for skewness and kurtosis. By using only the standard deviation as a measure of risk we implicitly assume that the distribution closely follows the normal trend.

Figure 1 shows the histogram (PDF) of the stock return (MSFT empirical) and the MSFT Required Final Value line. From this histogram we could easily find that the real distribution differs a lot in its general shape from the normal or any other exponential type distribution. It also has some local aberrations the so called psychological "resistance points" artificially set at almost every \$0.50. These points appear as a self fulfilling prophecy manifesting only as short bumps without much overall effect. Also due to these points the investment returns are even less likely to behave as random numbers.

Although the idea of using the standard deviation to represent the financial risk doesn't make much sense in a world where almost no investment follows the normal distribution this leads us to more generic views where we use for example the area under the PDF curve to represent the risk. An ideal return has a PDF described by a vertical line (zero risk) shifted towards the right as much as possible (high return).

Figure 1



Since the purpose of this paper is to derive the financial risk for the case of known events we assume now that before we made the investment we miraculously got access to the PDF of the stock return over the full 10 year investment period. We could also use the distribution to run a post-factum analysis. Either way if we define the financial risk as being the product between the potential outcome and the probability of that outcome we could easily find both the overall upward and downward potentials of this investment by summing (or integrating for a continuous distribution) all of these individual products described over the whole range of returns (in this particular case from $15 to $37). This method is very similar to the traditional way we determine costs in a statistical risk analysis and we'll call it the empirical risk measure.

The initial investment on Aug 1, 2000 was $26.33 and with a 2.5% target annual return rate the Required Final Value of the 10 year investment would be $33.70. Comparing this value with the ranges from Table 2 it becomes very clear that the investment is unlikely to be profitable but it remains difficult to suggest a value for the financial risk taken.

Let's estimate the empirical risk measure as defined above. If for example the value under analysis is $27.00 then the investment is a loss of $27.00 - $33.70 = -$6.70 per share. The corresponding probability of this event is 4.31% leading to an associated cost of -$6.70 * 0.0431 = -$0.2888. By computing the sum of all similar costs over the entire range we could determine that the downward potential was a loss of $7.264 and the upward potential a gain of $1.244 leading to an overall net potential loss of $6.020 per each share purchased as of 1 Aug 2000 with a 2.5% interest and additional costs. In Figure 1 all points to the right of the Required Final Value contribute to the profit and all points to the left contribute to the loss.

If we assume that the original investment was $26.33 and the interest rate zero, then the Required Final value is also $26.33 the downward potential (loss) is -$1.107 and the upward potential (gain) $1.244 or an overall net profit of $0.137. The net profit is not zero as expected because of the bumps and other sources of asymmetry in the empirical distribution.

Using this empirical technique it's easy to evaluate the real financial profit or loss for any investment irrespective of the statistical distribution. What's even more useful is the possibility of separating the loss and profit and selecting the Required Final Value based on some predetermined costs.

References

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